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The string dual of an $\mathcal{N} = (4,0)$ two-dimensional gauge theory

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ABSTRACT: We find solutions of the six-dimensional maximal supergravity by adding a perturbation of vector fields to the solution $AdS_3 \times \mathrm{S}^3$. For certain perturbations the solution represents a dual description of an $\mathcal{N} = (4,0)$ field theory in two dimensions by the AdS/CFT correspondence.

KEYWORDS: D-branes, AdS/CFT Correspondance, Supergravity Models.

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Contents

1. Introduction

One of the approaches to find field theories with a reduced supersymmetry in the context of the AdS/CFT correspondence [1, 2, 3] (for a review see ref. [4]) is to use the brane polarization [5]. Dp-branes are polarized by background antisymmetric tensor fields, even with the form degree larger than $p+1$. As an extension of the solution $AdS_5 \times \mathrm{S}^5$ in the ten-dimensional type-IIB supergravity, the authors of ref. [6] put a perturbation to it and found a new solution. By invoking the AdS/CFT correspondence, they argued that this solution is a dual of the $\mathcal{N}=4$ super Yang-Mills theory with a mass perturbation, which breaks the supersymmetry to $\mathcal{N}=1$.

In the ten-dimensional supergravity we try to find a large-N p-brane solution with a perturbation of electric RR $(p+4)$-form field strengths or its dual magnetic RR $(10-(p+4))$-form field strengths. Since the geometry of the large-N limit of p-brane solutions contains AdS_{p+2}, we perturb the AdS space with the $(6-p)$-form field strengths. In the D3-brane case [6] the RR 3-form field strengths are put in the directions of S^5, which is the partner of the AdS space. In the AdS/CFT correspondence the symmetry of the sphere is related to the R-symmetry of the field theory. When the R-symmetry is broken by the perturbation on the sphere, we have a field theory with a reduced supersymmetry. We expect that the rank of the field strengths for the perturbation should be half of the number of the sphere coordinates and the radial coordinate, that is $\frac{1}{2}(10-(p+2)+1)$. The field strengths have a background within the $(10-(p+2)+1)$-dimensional space $\mathrm{S}^{10-(p+2)} \times \mathbf{R}^+$. In the case of the eleven-dimensional supergravity compactified on $AdS_4 \times \mathrm{S}^7$ the branes are membranes $(p=2)$ and we perturb the solution by the 4-form field strengths within $\mathrm{S}^7 \times \mathbf{R}^+$.

In this paper we consider a perturbation to the solution $AdS_3 \times \mathrm{S}^3$ in the maximal supergravity in six dimensions [7], which is obtained from the ten-dimensional supergravity by a compactification on T^4. This solution is the geometry of the D1-D5 system. We put a perturbation of the RR field strengths of rank $4/2 = 2$ in S^3 and see how many supersymmetries are preserved by the perturbation. We find a perturbation that breaks half of the supersymmetries and gives a chiral theory in two dimensions. According to the original idea of the dielectric perturbation, the rank of the dual magnetic RR field strength in ten dimensions should be $10 - (1 + 4) = 5$. So let us consider that the rank 2 of the 5-form field strengths is in the sphere directions. We will discuss this perturbation in the supergravity in the last section.

The RR fields in the type-IIB supergravity are a scalar ϕ, a rank-2 antisymmetric tensor B_{MN} and a rank-4 B_{MNPQ}, where $M, N, \ldots = 0, 1, \ldots, 9$ are ten-dimensional world indices. If we compactify it on T^4, we have the six-dimensional maximal Poincaré supergravity [7]. Let us focus on the 4-form field, which plays the role of perturbation field in the procedure. We should choose B_{iIJK} for the field of the perturbation, where i denotes the radial coordinate of AdS_3 or a coordinate of S^3, and I, J, K denote coordinates of T^4. In other words, the 5-form field strength in ten dimensions for the perturbation appears as a 2-form field strength in the compactified six-dimensional theory.

The organization of this paper is as follows. In section 2 we give the solution $AdS_3 \times \mathrm{S}^3$ in six-dimensional supergravity. Then, we introduce a perturbation of gauge fields and find a solution for the field equations; we see how many supersymmetries are preserved by this perturbation in section 3. Section 4 is devoted to discussions about this procedure and our conclusion.

2. Compactification to three-dimensional AdS

Here we consider a solution for $AdS_3 \times \mathrm{S}^3$ in the six-dimensional Poincaré supergravity with the maximal supersymmetry [7]. This six-dimensional theory is obtained from the ten-dimensional type-IIB supergravity [8, 9] by a compactification on T^4.

The six-dimensional maximal supergravity has a rigid $\mathrm{SO}(5,5)$ symmetry and a local $\mathrm{SO}(5) \times \mathrm{SO}(5)$ symmetry. The field content of the theory is a sechsbein $e_M{}^A$, five antisymmetric tensor fields B^m_{MN}, 16 vector fields $A^{\tilde{\mu}\dot{\tilde{\mu}}}_M$, 25 scalar fields $\phi^{\alpha\dot{\alpha}}_{\tilde{\mu}\dot{\tilde{\mu}}}$, eight Rarita-Schwinger fields $\psi_{+\alpha M}$, $\psi_{-\dot{\alpha}M}$ and 40 spinor fields $\chi_{+a\dot{\alpha}}$, $\chi_{-\dot{a}\alpha}$, where the signs on the spinor fields denote the chiralities. The indices take values $m, a = 1, \ldots, 5$ and $\tilde{\mu}, \dot{\tilde{\mu}}, \alpha, \dot{\alpha} = 1, \ldots, 4$. The indices $\tilde{\mu}\dot{\tilde{\mu}}$ represent a spinor index of $\mathrm{SO}(5,5)$, while $a, \dot{a}$ and $\alpha, \dot{\alpha}$ represent vector and spinor indices of $\mathrm{SO}(5) \times \mathrm{SO}(5)$, respectively. The representation of each field for $\mathrm{SO}(5,5)$ and $\mathrm{SO}(5) \times \mathrm{SO}(5)$ is given in table 1. The

Fields	SO(5,5)	SO(5) × SO(5)
${e_M}^A$	**1**	**1**
(H^m_{MNP}, G^m_{MNP})	**10**	**1**
$A_M^{\tilde{\mu}\dot{\tilde{\mu}}}$	**16**	**1**
$\phi^{\alpha\dot{\alpha}}_{\tilde{\mu}\dot{\tilde{\mu}}}$	**16**	$(\mathbf{4}, \mathbf{4})$
$\psi_{+\alpha M}$	**1**	$(\mathbf{4}, \mathbf{1})$
$\psi_{-\dot{\alpha} M}$	**1**	$(\mathbf{1}, \mathbf{4})$
$\chi_{+a\dot{\alpha}}$	**1**	$(\mathbf{5}, \mathbf{4})$
$\chi_{-\dot{a}\alpha}$	**1**	$(\mathbf{4}, \mathbf{5})$

Table 1: The field content of the six-dimensional maximal supergravity.

field strengths of the antisymmetric tensor fields

$$H^m_{MNP} = 3\partial_{[M} B^m_{NP]} \tag{2.1}$$

and their duals G^m_{MNP} belong to **10** of SO(5,5).

We are only interested in the fields ${e_M}^A$, B^m_{MN} and $A_M^{\tilde{\mu}\dot{\tilde{\mu}}}$ and set other fields to zero except $\phi^{\alpha\dot{\alpha}}_{\tilde{\mu}\dot{\tilde{\mu}}} = \delta^\alpha_{\tilde{\mu}} \delta^{\dot{\alpha}}_{\dot{\tilde{\mu}}}$. By this background of the scalar fields the indices $\tilde{\mu}$ and α, $\dot{\tilde{\mu}}$ and $\dot{\alpha}$, and m and a are identified. The relevant part of the lagrangian is

$$\begin{aligned}\mathcal{L} = & \frac{1}{4} e_6 R - \frac{1}{12} e_6 H^a_{MNP} H^{aMNP} - \frac{1}{4} e_6 G^{\alpha\dot{\alpha}}_{MN} G^{MN}_{\alpha\dot{\alpha}} + \\ & + \frac{1}{2} e_6 G^{\alpha\dot{\alpha}}_{MN} A_{P\beta\dot{\alpha}} (\gamma^a)_\alpha{}^\beta H^{aMNP}_+ + \\ & + \frac{1}{2} e_6 G^{\alpha\dot{\alpha}}_{MN} A_{P\alpha\dot{\beta}} (\gamma^{\dot{a}})_{\dot{\alpha}}{}^{\dot{\beta}} H^{\dot{a}MNP}_- ,\end{aligned} \tag{2.2}$$

where $e_6 = |\det {e_M}^A|$ and γ^a are SO(5) gamma matrices. $H^a_{\pm MNP}$ denote the self-dual and anti-self-dual parts of H^a_{MNP}:

$$H^a_{\pm MNP} = \frac{1}{2} \left(H^a \pm *_6 H^a\right)_{MNP} , \tag{2.3}$$

where $(*_6 H^a)^{MNP}$ is the dual of H^a_{MNP}:

$$(*_6 H^a)^{MNP} = \frac{1}{6} e_6^{-1} \epsilon^{MNPQRS} H^a_{QRS} . \tag{2.4}$$

In this section we consider a unperturbed solution and set $A_M = 0$. The Einstein equation derived from eq. (2.2) is

$$R_{MN} - \frac{1}{2} g_{MN} R = H^a_{MPQ} H^a_N{}^{PQ} - \frac{1}{6} g_{MN} H^a_{PQR} H^{aPQR} . \tag{2.5}$$

By multiplying g^{MN} we see that the scalar curvature vanishes. The field equations and the Bianchi identities of B^m_{MN} are written as

$$\partial_M (e_6 H^{aMNP}) = 0 , \qquad \partial_M (e_6 (*_6 H^a)^{MNP}) = 0 . \tag{2.6}$$

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The metric for the compactification $AdS_3 \times \mathrm{S}^3$ is

$$\begin{aligned} ds^2 &= Z(r)^{-1} dx^\mu dx^\nu \eta_{\mu\nu} + Z(r) dx^i dx^j \delta_{ij} \\ &= Z(r)^{-1} dx^\mu dx^\nu \eta_{\mu\nu} + Z(r) dr^2 + R^2 d\Omega_3^2 \,, \end{aligned} \tag{2.7}$$

where the indices take values $\mu, \nu, \ldots = 0, 1$ and $i, j, \ldots = 1, 2, 3, 4$. The coordinates of the AdS space are x^μ and $r = (x^i x^i)^{1/2}$ and $d\Omega_3^2$ denotes the metric of a unit 3-sphere. $Z(r) = R^2/r^2$ is a harmonic function on $\mathbf{R}^4$, where R is the radius of the AdS space and S^3. In terms of the parameters of string theory, it can be written as

$$R^2 = \sqrt{Q_1 Q_5} g_6 \alpha' \,, \tag{2.8}$$

where Q_1 and Q_5 are the charges of the D1- and D5-branes, respectively, g_6 is the six-dimensional coupling constant, and α' is the Regge slope parameter. We take the following ansatz for the 3-form field strengths:

$$H^a_{\mu\nu i} = R^{-2} \mathcal{S}^a \epsilon_{\mu\nu} x_i \,, \qquad H^a_{ijk} = -r^{-4} R^2 \mathcal{T}^a \epsilon_{ijkl} x^l \,, \qquad \text{otherwise} = 0 \,, \tag{2.9}$$

where $\mathcal{S}^a$ and $\mathcal{T}^a$ are some constants. We find that eqs. (2.7) and (2.9) are solutions of the field equations (2.5) and (2.6) when $\mathcal{S}^a$, $\mathcal{T}^a$ satisfy

$$\mathcal{S}^a \mathcal{S}^a + \mathcal{T}^a \mathcal{T}^a = 2 \,. \tag{2.10}$$

The next task is to see how many supersymmetries are preserved by this solution. The local supersymmetry transformations of the spinor fields are [7]

$$\begin{aligned} \delta\psi_{+M\alpha} &= \hat{\mathcal{D}}_M \epsilon_{+\alpha} - \frac{1}{4} H^a_{+MNP} (\gamma_a)_\alpha{}^\beta \Gamma^{NP} \epsilon_{+\beta} \,, \\ \delta\psi_{-M\dot\alpha} &= \hat{\mathcal{D}}_M \epsilon_{-\dot\alpha} - \frac{1}{4} H^{\dot a}_{-MNP} (\gamma_{\dot a})_{\dot\alpha}{}^{\dot\beta} \Gamma^{NP} \epsilon_{-\dot\beta} \,, \\ \delta\chi_{+a\dot\alpha} &= \frac{1}{12} H^a_{+MNP} \Gamma^{MNP} \epsilon_{-\dot\alpha} \,, \\ \delta\chi_{-\dot a\alpha} &= \frac{1}{12} H^{\dot a}_{-MNP} \Gamma^{MNP} \epsilon_{+\alpha} \,. \end{aligned} \tag{2.11}$$

We decompose the six-dimensional gamma matrices as

$$\begin{aligned} \Gamma^A &= \hat\gamma^A \otimes \bar\gamma_{\mathrm{4D}} \,, \qquad (\text{for } A = 0, 1) \,, \\ \Gamma^A &= 1 \otimes \hat\gamma^A \,, \qquad (\text{for } A = 2, \ldots, 5) \,, \end{aligned} \tag{2.12}$$

where $\hat\gamma^A$ $(A = 0, 1)$ and $\hat\gamma^A$ $(A = 2, \ldots, 5)$ are gamma matrices of SO(1,1) and SO(4), respectively, and

$$\bar\gamma_{\mathrm{2D}} \equiv \hat\gamma^0 \hat\gamma^1 \,, \qquad \bar\gamma_{\mathrm{4D}} \equiv \hat\gamma^2 \cdots \hat\gamma^5 \,. \tag{2.13}$$

Inserting the solution (2.9) and (2.7) into (2.11) the conditions $\delta\chi_\pm = 0$ require

$$-\frac{1}{2r} x_i \bar\gamma_{\mathrm{2D}} \hat\gamma^i \left(\mathcal{S}^a \pm \mathcal{T}^a \right) \epsilon_\mp = 0 \,. \tag{2.14}$$

These equations are satisfied if we take $\epsilon_- = 0$ and $\mathcal{S}^a = \mathcal{T}^a$, or $\epsilon_+ = 0$ and $\mathcal{S}^a = -\mathcal{T}^a$. We choose the case $\epsilon_- = 0$ and $\mathcal{S}^a = \mathcal{T}^a$. In this case the antisymmetric tensor fields are self-dual $*_6 H^a_{MNP} = H^a_{MNP}$. Finally, we have to show the existence of a solution for $\delta\psi_{+M} = 0$ in eq. (2.11). We have checked that the integrability conditions for these equations

$$\left[\hat{D}_\mu - \frac{1}{2}\mathcal{S}^a(\gamma^a)\hat{\gamma}_\mu\bar{\gamma}_{\rm 2D}\otimes\hat{\gamma}_i x^i\bar{\gamma}_{\rm 4D}, \hat{D}_\nu + \frac{1}{4}\mathcal{S}^b(\gamma^b)\hat{\gamma}_\nu\bar{\gamma}_{\rm 2D}\otimes\hat{\gamma}_j x^j\bar{\gamma}_{\rm 4D}\right] = 0\,,$$

$$\Big[\hat{D}_\mu - \frac{1}{2}\mathcal{S}^a(\gamma^a)\hat{\gamma}_\mu\hat{\gamma}_{\rm 2D}\otimes\hat{\gamma}_l x^l\bar{\gamma}_{\rm 4D}, \hat{D}_i - \tag{2.15}$$

$$- r^{-2}\mathcal{S}^b(\gamma^b)\bar{\gamma}_{\rm 2D}\left\{1\otimes 1 x_i + \bar{\gamma}_{\rm 2D}\otimes\bar{\gamma}_{\rm 4D}\hat{\gamma}_{ik}x^k\right\}\Big] = 0\,,$$

$$\Big[\hat{D}_i - r^{-2}\mathcal{S}^a(\gamma^a)\bar{\gamma}_{\rm 2D}\left\{1\otimes 1 x_i + \bar{\gamma}_{\rm 2D}\otimes\bar{\gamma}_{\rm 4D}\hat{\gamma}_{ik}x^k\right\}, \hat{D}_j -$$

$$- r^{-2}\mathcal{S}^b(\gamma^b)\bar{\gamma}_{\rm 2D}\left\{1\otimes 1 x_j + \bar{\gamma}_{\rm 2D}\otimes\bar{\gamma}_{\rm 4D}\hat{\gamma}_{jl}x^l\right\}\Big] = 0 \tag{2.16}$$

are indeed satisfied by the above solution. Thus, we find that half of the supersymmetry corresponding to the parameters $\epsilon_{+\alpha}$ are preserved. This solution corresponds to an $\mathcal{N} = (4,4)$ conformal field theory in two dimensions [1, 10].

3. Three-dimensional AdS supergravity with perturbation

We consider a perturbation of the vector fields to the solution in section 2. The linearized field equations of the vector fields in the background (2.7) and (2.9) are

$$d *_6 G_2^{\alpha\dot{\alpha}} + 2G_2^{\beta\dot{\alpha}} \wedge H_3^a(\gamma^a)_\beta{}^\alpha = 0\,, \qquad dG_2^{\alpha\dot{\alpha}} = 0\,, \tag{3.1}$$

where $G^{\alpha\dot{\alpha}}_{MN}$ is the field strengths the vector fields $A^{\alpha\dot{\alpha}}_M$ and we have used the self-duality of H^a_{MNP}. The six-dimensional Hodge dual $*_6$ here can be expressed by the four-dimensional flat one $*_4$ as

$$*_6 G_2 = Z^{-1} *_4 G_2 \wedge dx^0 \wedge dx^1\,. \tag{3.2}$$

By using eqs. (2.7) and (2.9) the first equation of eq. (3.1) can be rewritten as

$$d\left[Z^{-1}\left(*_4 G_2^{\alpha\dot{\alpha}} + G_2^{\beta\dot{\alpha}}(\not{\mathcal{S}})_\beta{}^\alpha\right)\right] = 0\,, \tag{3.3}$$

where $\not{\mathcal{S}} = \mathcal{S}^a\gamma^a$. Since $\mathcal{S}^a\mathcal{S}^a = 1$, we have $\not{\mathcal{S}}^2 = 1$.

To solve these equations we introduce 2-forms with constant components $T_2^{\alpha\dot{\alpha}} = \frac{1}{2}T_{ij}^{\alpha\dot{\alpha}}dx^i \wedge dx^j$, which satisfy the (anti)self-duality condition

$$*_4 T_2^{\alpha\dot{\alpha}} = \pm T_2^{\alpha\dot{\alpha}}\,. \tag{3.4}$$

Their explicit forms are

$$T_2^{\alpha\dot{\alpha}} = m^{\alpha\dot{\alpha}} dz^1 \wedge d\bar{z}^2 + m^{\alpha\dot{\alpha}} d\bar{z}^1 \wedge dz^2 \tag{3.5}$$

for the self-dual case and

$$T_2^{\alpha\dot{\alpha}} = m^{\alpha\dot{\alpha}} dz^1 \wedge dz^2 + m^{\alpha\dot{\alpha}} d\bar{z}^1 \wedge d\bar{z}^2 \tag{3.6}$$

for the antiself-dual case, where $m^{\alpha\dot{\alpha}}$ is a constant matrix and z^1, z^2 are complex coordinates given by

$$z^1 = \frac{x^2 + ix^4}{\sqrt{2}}, \qquad z^2 = \frac{x^3 + ix^5}{\sqrt{2}}. \tag{3.7}$$

In the following we suppress the indices $\alpha\dot{\alpha}$, for simplicity unless necessary. Let us define

$$V_{ij} = \frac{x^k}{r^2}(x^i T_{kj} + x^j T_{ik}), \qquad S_i = T_{ij} x^j. \tag{3.8}$$

We can show the following relations:

$$\begin{aligned} dS_1 = 2T_2, \qquad dT_2 = 0, \qquad dV_2 = -2d(\ln r) \wedge T_2, \\ V_2 = d(\ln r) \wedge S_1, \qquad d(r^p S_1) = r^p(2T_2 + pV_2) \end{aligned} \tag{3.9}$$

and

$$*_4 V_2 = \pm(T_2 - V_2). \tag{3.10}$$

Using these tensors we can construct a solution to the linearized field equations (3.3) as follows. We make an ansatz

$$G_2 = \frac{1}{2} r^p (\alpha T_2 + \gamma V_2)(1 - \not{S}) + \frac{1}{2} r^q (\beta T_2 + \delta V_2)(1 + \not{S}). \tag{3.11}$$

The Bianchi identity $dG_2 = 0$ in eq. (3.1) gives

$$\gamma = \frac{p\alpha}{2}, \qquad \delta = \frac{p\beta}{2}, \qquad G_2 = \frac{1}{4}\alpha d(r^p S_1)(1 - \not{S}) + \frac{1}{4}\beta d(r^q S_1)(1 + \not{S}). \tag{3.12}$$

Using the duality properties (3.4) and (3.10) we have

$$\begin{aligned} *_4 G_2 - G_2 = {} & \frac{1}{4}\alpha r^p \left[(\pm p \pm 2 - 2)T_2 + (\mp p - p)V_2\right](1 - \not{S}) + \\ & + \frac{1}{4}\beta r^q \left[(\pm q \pm 2 + 2)T_2 + (\mp q + q)V_2\right](1 + \not{S}). \end{aligned} \tag{3.13}$$

The field equation in (3.3) then gives

$$p^2 + 6p + (4 \mp 4) = 0, \qquad q^2 + 6q + (4 \pm 4) = 0. \tag{3.14}$$

Let us first consider the α terms. For the lower sign, there are two solutions of eq. (3.14):

$$
\begin{aligned}
p &= -4\,; \qquad G_2 = \frac{\alpha}{2} r^{-4} (T_2 - 2V_2)(1 - \not{S})\,, \\
p &= -2\,; \qquad G_2 = \frac{\alpha}{2} r^{-2} (T_2 - V_2)(1 - \not{S})\,.
\end{aligned} \tag{3.15}
$$

For the upper sign, there are also two solutions of eq. (3.14):

$$
\begin{aligned}
p &= -6\,; \qquad G_2 = \frac{\alpha}{2} r^{-6} (T_2 - 3V_2)(1 - \not{S})\,, \\
p &= 0\,; \qquad G_2 = \frac{\alpha}{2} T_2 (1 - \not{S})\,.
\end{aligned} \tag{3.16}
$$

Similarly, the solutions for the β terms are

$$
\begin{aligned}
q &= -6\,; \qquad G_2 = \frac{\beta}{2} r^{-6} (T_2 - 3V_2)(1 + \not{S})\,, \\
q &= 0\,; \qquad G_2 = \frac{\beta}{2} T_2 (1 + \not{S})
\end{aligned} \tag{3.17}
$$

for the lower sign and

$$
\begin{aligned}
q &= -4\,; \qquad G_2 = \frac{\beta}{2} r^{-4} (T_2 - 2V_2)(1 + \not{S})\,, \\
q &= -2\,; \qquad G_2 = \frac{\beta}{2} r^{-2} (T_2 - V_2)(1 + \not{S})
\end{aligned} \tag{3.18}
$$

for the upper sign.

Let us count how many supersymmetries are preserved by this perturbation. The additional terms to the supertransformations generated by the perturbation are

$$
\begin{aligned}
\delta\psi_{-\mu\dot{\alpha}} &= \frac{1}{8} i G_{jk\beta\dot{\alpha}} (\hat{\gamma}_\mu \times \hat{\gamma}^{jk}) \epsilon_+^\beta\,, \\
\delta\psi_{-i\dot{\alpha}} &= -\frac{3}{4} i G_{ij\beta\dot{\alpha}} (\mathbf{1} \times \hat{\gamma}^j) \epsilon_+^\beta + \frac{1}{8} i G_{jk\beta\dot{\alpha}} (\mathbf{1} \times \hat{\gamma}_i{}^{jk}) \epsilon_+^\beta\,, \\
\delta\chi_{+a\dot{\alpha}} &= \frac{1}{4} G_{ij\beta\dot{\alpha}} (\mathbf{1} \times \hat{\gamma}^{ij}) \epsilon_+^\alpha (\gamma_a)_\alpha{}^\beta\,.
\end{aligned} \tag{3.19}
$$

Generically, these terms do not vanish and supersymmetry is completely broken. However, for particular backgrounds $G_{ij\beta\dot{\alpha}}$ some of the supersymmetries are preserved. Let us consider the α terms with $p = -4$ for the lower sign or those with $p = 0$ for the upper sign, in which $G_{ij\beta\dot{\alpha}}$ is self-dual. If we impose the condition $\bar{\gamma}_{4D}\epsilon_+ = \epsilon_+$, $\delta\psi_{-\mu\dot{\alpha}}$ and $\delta\chi_{+a\dot{\alpha}}$ are shown to vanish by using the identity $\hat{\gamma}^{ij} = -\frac{1}{2}\epsilon^{ijkl}\hat{\gamma}_{kl}\bar{\gamma}_{4D}$. This condition does not guarantee the vanishing of $\delta\psi_{-i\dot{\alpha}}$. Since it is proportional to $\epsilon^\alpha m_{\alpha\dot{\alpha}}$, the remaining supersymmetries are determined by zero eigenvalues of the matrix m. For instance, let us consider the following form of the matrix

$$
m_{\alpha\dot{\alpha}} = \frac{1}{2} m^{mn} (\gamma_{mn})_{\alpha\dot{\alpha}}\,. \tag{3.20}
$$

Here, γ_{mn} is an antisymmetrized product of gamma matrices for the SO(4) subgroup in SO(5), which is chosen such that the chirality matrix of SO(4) is $\not{S}$. Then,

$\delta\psi_{-i\dot{\alpha}}$ vanishes when m_{mn} is self-dual. Since $\bar{\gamma}_{4D}\epsilon_+ = \epsilon_+$ means $\bar{\gamma}_{2D}\epsilon_+ = \epsilon_+$ for six-dimensional spinors ϵ_+ of positive chirality, $\mathcal{N} = (4,0)$ supersymmetry in two dimensions is preserved in this case.

4. Summary and conclusion

We found a supergravity solution with the reduced supersymmetry, which is the dual string theory of an $\mathcal{N} = (4,0)$ field theory. We discussed the following.

First, we constructed an $\mathcal{N} = (4,0)$ field theory from the point of view of supergravity. We should also discuss it from the field theory side. We need to find an operator corresponding to the perturbation in our solution. From the r-dependence of the perturbation we can read off the conformal dimension of the operator. It is also an interesting problem to find out a holographic renormalization group flow from $\mathcal{N} = (4,4)$ to $\mathcal{N} = (4,0)$ in two-dimensional field theory in the context of the supergravity.

Second, we are interested in how can be interpreted the puffing up of the D-branes in $S^3 \times T^4$. Since we did not put the antisymmetric tensor in the whole space of the sphere S^3, the polarization does not seem to take place. But if we consider T^4 in addition to the sphere, we will see the polarization, but we need to consider the topology there.

Third, we considered the gauge field potentials, which are related to the D1-branes. But we did not consider the precise effects of the D5-branes when we discussed the perturbation. In our work, the hidden three directions belong to the four directions in the compactified manifold T^4, where the D5-branes also exist. The torus T^4 should be treated carefully. At least in the six-dimensional $AdS_3 \times S^3$ space, we see the RR charge is in S^1, just like the 2-forms are in S^2 for the case of large-N D3-branes [6].

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